Electrons, gammas and pi0, in heavy liquid bubble chambers

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ELECTRONS, GAMMAS AND π^o, IN HEAVY LIQUID BUBBLE CHAMBERS

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1. INTRODUCTION

Electromagnetic, weak and strong interactions studies, involve electron trajectories in form of isolated electrons or photon-materializa tion pairs, many of them coming indirectly from the well-known $\pi^o \rightarrow 2\gamma$ disintegrations.

At least in principle, the study of reactions or disintegratio giving nearly only e, γ or π^o is made possible by use of heavy liquid bubble chambers. The probability for detection of the 3 mentioned proce has to be made as high as possible. But then, a conflict appears betwee high detection and good measurability of electrons. An optimum is to be found, generally in favour of detection. In such a situation, it is nec sary to improve as much as possible the quality and the reliability of electrons measurement techniques.

The behaviour, in the physical medium of the chamber, of elect is quite different from the one of ordinary particles: mainly because it very low rest-mass. Radiation effects occur with a net non-zero mean va (while effect such as scattering gives on the trajectories a null mean perturbation). Consequently, the path of an electron along the chamber no longer be identified with a circle (preferably a cylindrical helix).

The solution to this problem will receive new applications in giant chambers for which energy losses by collision become important so that also "circular" approximation is always unsatisfactory.

The aim of this paper is to give necessary basis for understan how to measure, calculate and use electronic trajectories and how to com- bine them in γ or π^o.

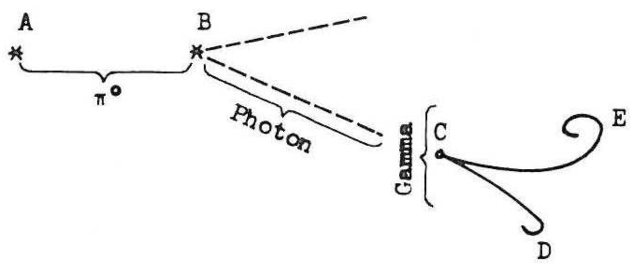
Some related topics are also briefly reviewed such as δ-rays and radiation length determination.

In no case, the present text is exhaustive. It tries only to be explicit for the physicists non- familiar with heavy liquid bubble chambers.

Caution

For convenience, we shall use, along these notes, the words

- "gammas" for assemblies (e^+,e^-) after photon-materialization (CE + CD)
- "photon" for the neutral line of flight BC before materialization.



In supplement, abbreviation H.L.B.C. must be often used for heavy liquid bubble chamber.

2. THEORETICAL ASPECTS

In this chapter will be recalled the different effects occurrin to electrons and photons when they pass through matter (chamber liquid). In addition, the π^o-disintegration process will be considered.

2.1 Electrons and Material Medium

Three well-known effects can appear when a charged particle traverses matter:

1) Electromagnetic interaction between the incident charge and the charges of peripherical electrons of medium atoms. For electrons, special formula (modified Bethe-Bloch formula) must be developed (essentially because, first "bullet" and target have the same mass, and secondl they are not distinguishable) for this collision process (ionization).

2) Electromagnetic interaction between the incident charge and the charges of the medium nuclei.

3) When the distance of closest approach becomes of the order the atomic radius, the deflection of the incident particle results in the emission of quantas. Quantum - theoretically "soft" quantas are more probably emitted than energetic ones. It is thus possible to disconnect the problem of only angular deviation (scattering) from that of the emiss (radiation or bremsstrahlung)

2.1 a) Energy Loss by Collision or Ionization

Let -dE/dx be the density of energy loss per unit length. The general Bethe-Bloch formula related to this process is written, in general[1)]

$$- \frac{1}{\rho}\frac{dE}{dx} = \frac{2\pi\ z^2\ e^4}{m_e\ v^2}\frac{N_e}{\rho}\left\{ \mathrm{Ln}\ \frac{2\ m_e\ v^2\ W_{max}}{I^2(1-\beta^2)} - 2\beta^2 - 2\ \frac{c}{Z} - \delta\right\} \qquad (2\text{-}1\text{-}1)$$

where

$$\frac{e^4}{m_e} = r_e^2\ m_e = (2.82\ \text{fermi})^2\ (0.511\ \text{MeV})$$

ρ = density in gr cm^{-2} Z = atomic number A = mass number

$N_e = N_o \rho \frac{Z}{A}$ = density of electrons

z = incident charge = 1 for electrons

I = experimental mean ionization potential for (A,Z) element.

$W_{max} = \frac{1}{2} T_e$ = (kinetic incident energy)/2

v = incident velocity $\beta = v/c$ $\gamma = (1 - \beta^2)^{-1/2}$

The (c/Z)-term takes into account "shells-correction" to make the formula correct for low β-value.[2)]

The δ-term corresponds to a "density-correction" including polarization effects inside the medium for high β-value.[3)]

Since for an electron $\beta \simeq 1$, dE/dx can be considered as constant For a mixture the combination law is

$$\left(\frac{1}{\rho} \frac{dE}{dx}\right)_{Mix} = \sum_j \left(\frac{1}{\rho_j} \frac{dE}{dx}\right)_j P_j \qquad (2\text{-}1\text{-}1 \text{ bis})$$

where P_j is the fractional weight of element $(A,Z)_j$ and ρ_j the partial density.

Table 2-1/1 gives value of I, c/Z and δ for different values of (A,Z).

Table 2-1/1

Material	Z	A	I	c/Z	δ	Chamber Liquids
H	1	1	19.05	0		C^3H^8
D	1	2	19.05	0		
He	2	4	48.1	0		
C	6	12	78.0	.0010	∿ 1.	C^3H^8, C^2F^5Cl, CF^3Br
F	9	19	117.2	.0014	∿ 1.5	C^2F^5Cl, CF^3Br
Ne	10	20	130.0	.0017	∿ 1.7	Mixture (Ne + H)
Cl	17	35.5	217.6	.0037	∿ 3.	C^2F^5Cl
Br	35	80	430.0	.0104	∿ 3.	CF^3Br
I	53	127	689.0	.0189	3.5	ICH^3

β → 1

Ranges of electrons can not be deduced only by integrating the Bethe-Bloc formula (2-1-1), the most important energy-loss coming from radiation (se below 2-1-c).

2.1 b) Energy Loss per Radiation (Bremsstrahlung)

As stressed before, emission of large quanta plays an important part in energy loss for an electron traversing matter. If the particle loses its energy in the form of a large number of light quantas, the effects of fluctuations on the mean rate of energy loss by collision (calle straggling in the case of heavy particles) must be very small, but if som hard quantas have been emitted, straggling will be very large.

Neglecting the classical energy loss per collision, one can evaluate the energy distribution law for typical radiation process.

Assuming a Thomas-Fermi model for the atom, the frequency for the emitted photons is[2)]

$$\phi_k \, d\left(\frac{k}{E_o}\right) = 2 \, \langle\phi\rangle \, \frac{dk}{k} \, \frac{E}{E_o} \left\{ \left(\frac{E_o^2+E^2}{E_o E} - \frac{2}{3} \right) 2 \, \mathrm{Ln} \, (183 \, Z^{-1/3}) + \frac{2}{9} \right\} \qquad (2-$$

where $k = \hbar\nu$ = energy of emitted light quantum

E = actual energy of incident electron

E_o = primary energy of incident electron

$E_o - \mu$ = primary kinetic energy of incident electron

ϕ_k = cross section for the emission of quantum in the range of $(k, k + dk/(E_o-\mu))$

Ln = Neparian logarithm of

so that, the average energy loss per cm is given by

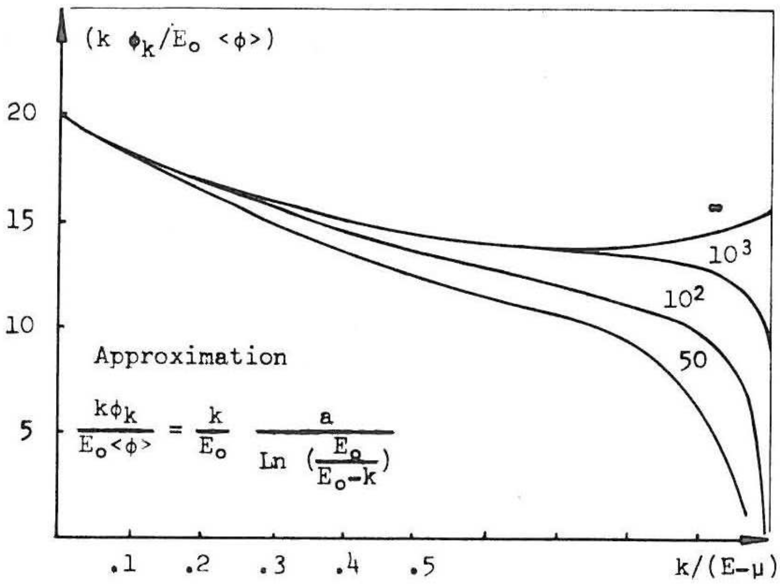
$$- \frac{dE_o}{dx} = N \int_0^1 k \, \phi_k \, d\left(\frac{k}{E_o-\mu}\right) \qquad (2\text{-}1\text{-}3)$$

with

N = number of atoms/cc and $\mu = m_e c^2$

Figure 2-1/1 shows the graphical representation of equation (2-1-2)

Figure 2-1/1



Number attached to the curves indicates $(E_o-\mu)/\mu$ for the incident electron.

a) for $E_o \gtrsim 25$ MeV, area under curves are of the same order

b) therefore

$$- \frac{dE_o}{dx} \cdot \frac{1}{E_o} \approx \text{constant}$$

and

$$- \frac{dE_o}{dx} = NE_o\ \phi_{rad} \tag{2-1-4}$$

with

$$\phi_{rad} = \frac{1}{E_o} \int_0^1 k\ \phi_k\ d(\frac{k}{E_o-\mu})$$

To approximate the general features of Figure (2-1/1), one can write[3)] that the probability $\overline{\omega}(k)\ dk$, that the electron has lost an energy k for the infinitesimal path dx, is

$$\overline{\omega}(k)\ dk = \frac{a <\phi> N}{E_o} \frac{dk\ dx}{Ln\ (\frac{E_o}{E_o-k})} \qquad (2\text{-}1\text{-}5)$$

By a variable transformation $y = Ln \frac{E_o}{E_o-k} = Ln \frac{E_o}{E}$, with the new constant

$$b = a <\phi> N \rightarrow E = E_o - k = E_o \exp(-y)\ ;\ \overline{\omega}(k)\ dk = b\ dx\ \frac{e^{-y}\ dy}{y}$$

and

$$\phi_{rad} = \frac{N}{E_o^2} \int_o^{E_o} k\ \phi_k\ dk = b \int_o^{\infty} \frac{e^{-y}(1-e^{-y})}{y}\ dy = b\ Ln\ 2 \qquad (2\text{-}1\text{-}6)$$

In conclusion, for dx, the probability that the incident energy has decreased by a factor e^{-y} is

$$\overline{\omega}(dx,y)\ dy = b\ dx\ \frac{e^{-y}\ dy}{y} = \frac{e^{-y}\ y^{bdx-1}}{\Gamma(bdx)}\ dy \qquad (2\text{-}1\text{-}7)$$

b is a constant characteristic of the crossed medium (see below (2-1-3)) y is simply related to the energy loss by radiation. Γ is the well-known "gamma function".

(2-1-7) has the same shape as a χ^2-distribution. As we know that the χ^2-distribution of the sum of two normal variables is also a χ^2-distribution, we can immediately deduce from this property that (2-1-7) is also valid for finite range x. Thus

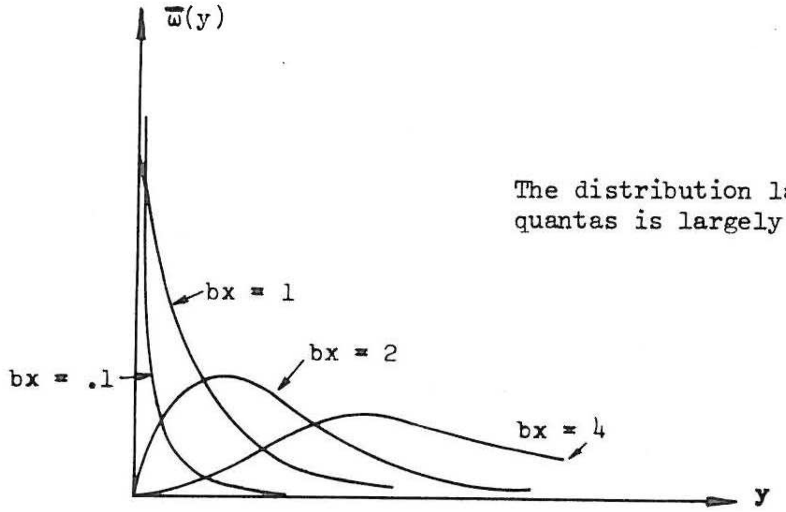
$$\overline{\omega}(x,y)\ dy = \frac{e^{-y}\ y^{bx-1}}{\Gamma(bx)}\ dy \quad \text{(see fig. 2-1/2)} \qquad (2\text{-}1\text{-}7bi$$

THIS DISTRIBUTION IS ONLY FUNCTION OF E_o/E AND NOT OF E_o (as long as $X_o(E_o)$ = constant). Using classical relation: cross section . mean free path = unity, we have

$$\phi_{rad} \cdot X_o = 1 \quad \text{or} \quad b = \frac{1}{X_o\ Ln\ 2} \qquad (2\text{-}1\text{-}8)$$

X_o is called radiation length for the considered compound (see below 2-1-c), that is the distance over which the electron has its energy divid (in mean) by a factor e.

Figure 2-1/2



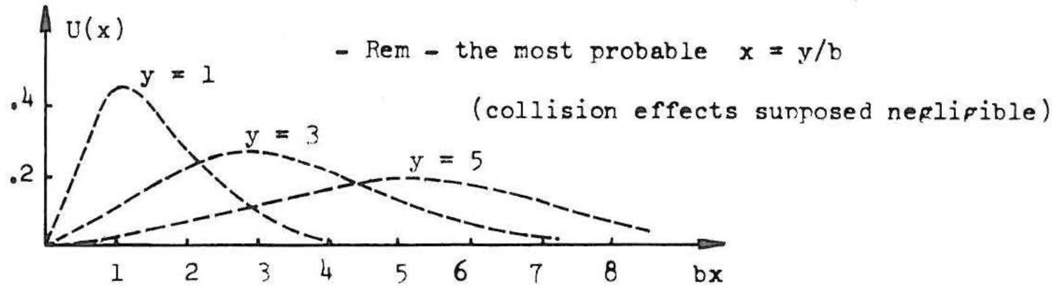
The distribution law of different emitted quantas is largely disymmetrical.

We can now reconsider the range of an electron. The chance that after travelling a distance x, an electron, with initial energy E_o, has an energy still > E, is given by $\overline{\omega}(bx,y)$ with $y = Ln\ (E_o/E)$. This cha after $x + dx$, is $\overline{\omega}(bx + bdx,\ y)$. Thus the probability that the energy ha: crossed the E value between x and $x + dx$ is

$$U(x)dx = \overline{\omega}(bx,y) - \overline{\omega}(bx + bdx,\ y) = -\ \frac{\partial\overline{\omega}(bx,y)}{\partial x}\ dx$$

$U(x)dx$ is the distribution law of the range x, for a fixed y (see figure 2-1/3).

Figure 2-1/3



In conclusion, energy determination by range measurement is exposed to very large errors (nonsymmetrical).

A numerical example for illustration:

$X_o = 25$ cm $(C_2F_5Cl) \rightarrow b \simeq 1/15$

$E_o = E.20 \rightarrow y = 3$ and $1 \leqslant bx \leqslant 5$ so $15 \leqslant x \leqslant 75$ cm

and

the most probable x has the value 45 cm.

Another interesting question is: which is the direction of emiss of radiated quantas ? To answer this question, one can use a semi-quantitative argument. Let θ' be the emission angle of a quantum in the rest-system of the electron, the corresponding angle θ in the lab-system is given by

$$\theta = \sqrt{1-\beta^2}\ \frac{\sin\theta'}{1+\cos\theta'} \qquad (\beta \simeq 1)$$

as

$$<\left|\frac{\sin\theta'}{1+\cos\theta'}\right|> \simeq 1 \;\rightarrow\; <|\theta|> \simeq \sqrt{1-\beta^2} = \frac{m_e\, c^2}{E} \qquad (2\text{-}1\text{-}9)$$

Thus θ becomes very small with increasing E. Practically as $m_e\, c^2 = 0.5$ and $E_o \geqslant 50 \rightarrow <|\theta|> < 1$ milliradian $\approx .5$ degree.

In general, one can make the assumption that all quantas of bremsstrahlung are emitted along the tangent to the trajectory of the emitting electron. As a consequence, the dip of an electron-track is not affected by radiation process.

2.1. c) Radiation Length Calculations

Inside X_o is included the medium. Because we are interested in electrons whose energy is at least 25 MeV, the formula giving X_o is[3]

$$\frac{1}{X_o} = \frac{4}{137}\ \frac{N}{A}\ Z(Z+\xi)\ r_e^2\ \mathrm{Ln}\left(\frac{183}{Z^{1/3}}\right) \qquad (2\text{-}1\text{-}10)$$

$(E_o >> 137\ m_e\, c^2\ Z^{-1/3} \simeq 25 \qquad <Z> \sim 15$ in general)

with N = number of atoms (Z,A) per cc
ξ = correction to take into account the screening effects of atomic electrons.

For a mixture, the composition law is

$$\frac{1}{X_o} = \sum \frac{p_i}{(X_o)_i} \quad \text{with } p_i = \text{fractionnal weight of component } (A,Z)_i \qquad (2\text{-}1\text{-}10\text{bis})$$

Table 2-1/2 gives X_o (cm) and density (gr/cc) for various subtances related to HBLC liquids.

Table 2-1/2

	Z	A	ρ	X_o	ξ
H_2	1	1	.0628	915	1.39
He	2	4	.12	750	
C	6	12	1.55	27.3	1.32
F	9	19	1.11	29.5	1.30
Ne	10	20	1.2	24	
Cl	17	35.5	1.56	12.3	1.26
Br	35	80	3.1	3.61	1.22
I	53	127	4.93	-	1.19
Xe	54	131	3.52	3.5	
C_3H_8			.41	112	
C_2F_5Cl			1.2	24.2	
CF_3Br			1.5	10.6	
ICH_3					

2.1 d) Scattering Effect[4)]

If the electron has a constant momentum p_o, the scattering angu lar variance after a traversal x is

$$\sigma_\phi^2 = <\phi^2> - <\phi>^2 = <\phi^2> = \frac{1}{2} \frac{E_s^2}{p_o^2 X_o} x = \frac{1}{2} \Phi_o^2 x \qquad (2\text{-}1\text{-}11)$$

where

E_s has a value 21 MeV

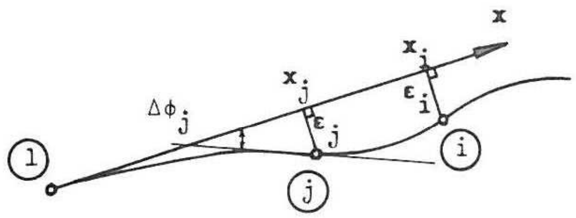
X_o is the radiation length in cm.

The $\frac{1}{2}$-factor comes from the fact that, in the precedent formula, we consi only a plane-projection of the trajectory (in space, this factor must be omitted).

Φ_o^2 is called scattering constant. In general, the kinematic variable in denominator is $(p_o\beta)$ but here $\beta \simeq 1$.

For establishing (2-1-11), p_o is supposed fixed to its initial value. This condition is never reached for the electron 1) because the collision energy loss 2) because the radiation which causes very sudden and large energy drop. For these reasons it is necessary to modify (2-1- We will re-establish formulas, not for angular, but for lateral displacem with regard to the initial tangent (which are more convenient in practice

At first, fixing p to value p_o, we can write for lateral displa ments ε, the relation



$$\varepsilon_i = \varepsilon_j + (x_i - x_j)\,\Delta\phi_j + \varepsilon_{ji}$$

with ε_{ji} = lateral displacements f the path $(x_i - x_j)$.

Then

$$<\varepsilon_i\,\varepsilon_j> = <\varepsilon_j^2> + (x_i - x_j)\,<\varepsilon_j\,\Delta\phi_j>$$

taking into account the statistical independence between ε_i and ε_{ij} and $<\varepsilon_i> = <\varepsilon_{ij}> = 0$.

For the value of $<\varepsilon_j \phi\Delta_j>$, one obtains[4] $<\varepsilon_j \Delta\phi_j> = \frac{1}{4}\phi_o^2 x_j^2$

$$<\varepsilon_i \varepsilon_j> = \sigma_{ij}^2 = \frac{1}{12}\phi_o^2 x_j^2 (3x_i - x_j) \qquad (2\text{-}1\text{-}12)$$

(2-1-12) gives all the scattering matrix elements (p constant).

Now consider the case where there is a mean constant variation of p along the trajectory. One has

$$\varepsilon_1 = 0$$

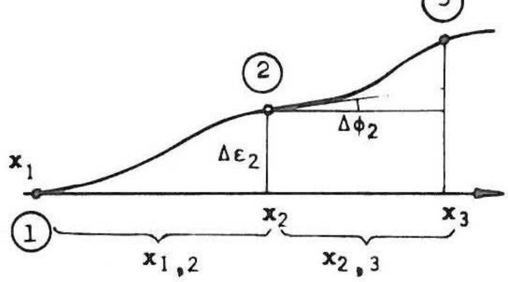
$$\varepsilon_2 = \Delta\varepsilon_2 + \Delta\phi_1 \; x_{1,2} = \Delta\varepsilon_2$$

$$\varepsilon_3 = \Delta\varepsilon_2 + \Delta\phi_2 \; x_{2,3} + \Delta\varepsilon_3 = \varepsilon_2 + \Delta\phi_2 \, x_{2,3} + \Delta\varepsilon_3$$

$$\varepsilon_4 = \varepsilon_3 + \Delta\phi_3 \; x_{3,4} + \Delta\varepsilon_4$$

.....

$$\varepsilon_j = \sum_{k=1}^{j-1} \left[(1 + \frac{3}{2} \frac{x_{k+1,j}}{x_{k,k+1}}) \Delta\varepsilon_{k+1} + x_{k+1,j} \Delta\phi_{k+1} \right]$$



We know from normal scattering formulas that

$$\left.\begin{aligned} <\Delta\varepsilon_{j+1} \Delta\varepsilon_{\rho+1}> &= \frac{1}{6} \frac{E_s^2}{X_o} x_{j,j+1}^{3/2} \; x_{\rho,\rho+1}^{3/2} \frac{\delta_{j\rho}}{(p\beta c)_j^2 (p\beta c)_\rho^2} \\ <\Delta\phi_{j+1} \Delta\phi_{\rho+1}> &= \frac{1}{8} \frac{E_s^2}{X_o} x_{j,j+1}^{1/2} \; x_{\rho,\rho+1}^{1/2} \frac{\delta_{j\rho}}{(p\beta c)_j^2 (p\beta c)_\rho^2} \end{aligned}\right\} \quad \delta_{j\rho} = \begin{cases} 1 \text{ si } j = \rho \\ 0 \text{ si } j \neq \rho \end{cases}$$

$$\sigma_{ik}^2 = \langle \varepsilon_i \; \varepsilon_k \rangle = \sum_{j=1}^{i-1} \sum_{\rho=1}^{k-1} \left[\left(1 + \frac{3}{2} \, \frac{x_{j+1,i}}{x_{j,j+1}}\right)\left(1 + \frac{3}{2} \, \frac{x_{\rho+1,k}}{x_{\rho,\rho+1}}\right) \langle \Delta\varepsilon_{j+1} \; \Delta\varepsilon_{\rho+1} \rangle \right.$$

$$\left. + \; x_{j+1,i} \;\; x_{\rho+1,k} \;\; \langle \Delta\phi_{j+1} \; \Delta\phi_{\rho+1} \rangle \right]$$

$$= \frac{E_s^2}{6X_o} \sum_{j=1}^{i-1} \left\{ \left[\left(1 + \frac{3}{2} \, \frac{x_{j+1,i}}{x_{j,j+1}}\right)\left(1 + \frac{3}{2} \, \frac{x_{j+1,k}}{x_{j,j+1}}\right) x_{j,j+1}^3 + \frac{3}{4} \, x_{j+1,i} \;\; x_{j+1,k} \right] \frac{1}{p_j^2} \right.$$

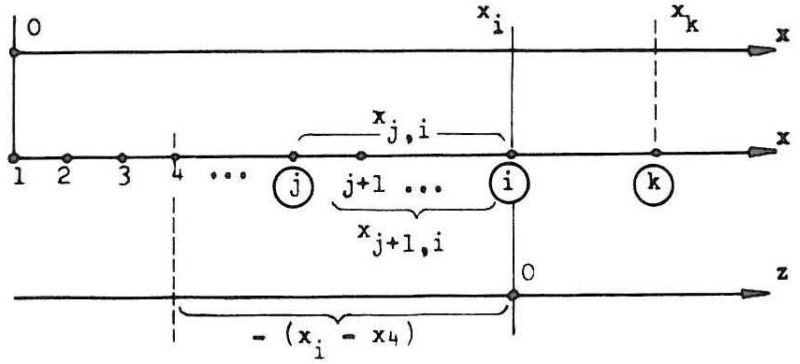
- making the transformations $x_{j+1,k} = x_{j+1,i} + x_{i,k}$:

$$\sigma_{ik}^2 = \frac{E_s^2}{6X_o} \left\{ \sum_{j=1}^{i-1} \left(\frac{1}{p_j^2} \left[(x_{j,i} - x_{j+1,i})(x_{j,i}^2 + x_{j,i} \; x_{j+1,i} + x_{j+1,i}^2) \right] \right) \right.$$

$$\left. + \; \frac{3}{2} \sum_{j=1}^{i-1} \left(\frac{1}{p_j^2} \left[(x_{j,i} - x_{j+1,i})(x_{j,i} + x_{j+1,i}) \right] \right) \right\} \qquad \text{(2-1-13)}$$

- up to the limits

$$j \to \infty \qquad \begin{cases} x_{j,i} = x_{j+1,i} = -z \\ x_{j,i} - x_{j+1,i} = dz \end{cases}$$



We have - for $z = -x \rightarrow$ first point $\rightarrow p_1 = p_o$

- for $z = 0 \rightarrow$ i point $\rightarrow p_i = f(x_i)p_o \rightarrow \frac{1}{p_i^2} = g(x_i)\frac{1}{p_o^2}$

Therefore

$$\sigma^2_{x_i x_j} = \lim_{k\to\infty} \sigma^2_{ij} = \frac{\phi_o^2}{2}\left\{\int_{-x_i}^{o} z^2\, g(z + x_i)dz - (x_j - x_i)\int_{-x_i}^{o} z\, g(z + x_i)dz\right\} \tag{2-1-14}$$

In general, the two integrations are easy. We will consider in future, two applications of (2-1-14).

(2-1-13) includes all point-to-point scattering correlations, even those having an external (energy loss) origin.

In the same manner, it is possible to deduce the formulas for $<(\Delta\phi_j)^2>$ and $<\varepsilon_j\ \Delta\phi_j>$.

2.2 Photons and Material Medium

From the point of view of usefulness, there are two kinds of photons: first those emitted in interactions or disintegrations; for them it is necessary to estimate their kinematical (momentum) and geometrical (angles) parameters; secondly, the quantas of radiation, which are spurious photons and have in general no utility.

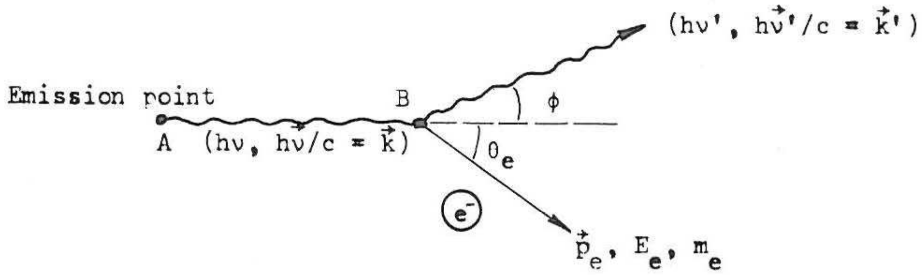
"Measurable" photons are in fact known by their secondary products which are of two classes: gammas and compton electrons.

2.2 a) Compton Effect

This phenomenon is strictly identical to the collision-process for other particles, the incident charge excepted.

* This formula has been established in collaboration with P. Schliapnikov of the Joint Institute for Nuclear Research (Dubna, US.S.R.)

Remembering that a proton whose energy is $h\nu$ has momentum $h\nu/c$, we can write the energy-momentum conservation laws



$$k'^2c^2 = p_e^2\, c^2 + k^2\, c^2 - 2\, p_e\, kc^2 \cos\theta_e$$

$$k'c + m_e\, c^2 = -\left[(p_e c)^2 + m_e^2\, c^4\right]^{1/2} + kc$$

Eliminating $k'c$ and writing $Q = kc/m_e c^2$, $q = p_e c/m_e c^2$

$$Q = \frac{(1 + q^2)^{1/2} - 1}{1 + q\cos\theta_e - (1 + q^2)^{1/2}} \qquad (2\text{-}2\text{-}1)$$

If $\begin{cases} \theta_e = 0 \rightarrow p_e^{max} = \dfrac{2Q(Q+1)}{(2Q+1)} & \text{(a little larger than } Q) \\ \theta_e = \dfrac{\pi}{2} \rightarrow p_e^{min} = 0 & \end{cases}$

All θ_e values are not equally probable. The probability of Compton effect decreases very rapidly when θ_e increases. The angular distribution of the recoil electron is given by a complex formula which can be approximated under the assumption $(Q \gg 1)$ by

$$\frac{d\phi}{r_e^2\, d\Omega_{\theta_e}} = \frac{Q}{Q^2\, \theta_e^2 + 1} \quad \text{if} \quad (Q\, \theta_e^2 \ll 1) \qquad (2\text{-}2\text{-}2)$$

or

$$\frac{d\phi}{r_e^2\, d\Omega_{\theta_e}} = \frac{4\cos\theta_e}{Q^2\sin^4\theta_e} \quad \text{if} \quad (Q\sin^2\beta \gg 1) \qquad (2\text{-}2\text{-}3)$$

($d\Omega_{\theta_e}$ = unit solid angle in the recoil electron direction)

The higher the incident energy, the narrower the angular distribution. One can also derivate the probability of scattering for quantas as a function of their energy. For $Q >> 1$, one obtains in terms of cross section

$$(Q >> 1) \qquad \Pi = \pi r_e^2 \frac{1}{Q} (\text{Ln } 2Q + \frac{1}{2}) \tag{2-2-4}$$

Let us remark that Π is independent of the medium and is inversely proportionnal to the quantum energy.

2.2 b) <u>Pair Creation</u>

In nuclei, high-energy photons are absorbed and form electron-positron pairs in the electromagnetic field of the nucleus. These pairs are commonly named "gammas" by HLBC physicists. If we simply consider γ having energy larger than ~ 50 MeV, the screening effect of atomic electrons is complete. Then the cross section for pair production can be written

$$S = \frac{Z^2}{137} r_e^2 \left\{\frac{28}{9} \text{Ln } 183 Z^{-1/3} - \frac{2}{27}\right\} \tag{2-2-5}$$

S is usually expressed in terms of $1/X_o$; we can then define a "pair creation length" C_o which is slightly larger than the radiation length. The decrease of the intensity n of a beam of photons will follow the equation

$$\frac{dn}{n} = - \frac{dx}{C_o} \rightarrow n = n_o \exp\left\{- \frac{x}{C_o}\right\}$$

with

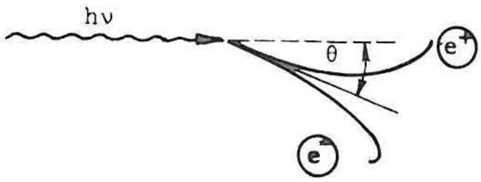
$$C_o \simeq \frac{1}{\frac{28}{9} (\text{Ln } 183 Z^{-1/3}) \frac{Z^2N}{137}} \tag{2-2-6}$$

Comparing with (2-1-9) :

$$C_o = \frac{9}{7} X_o \tag{2-2-7}$$

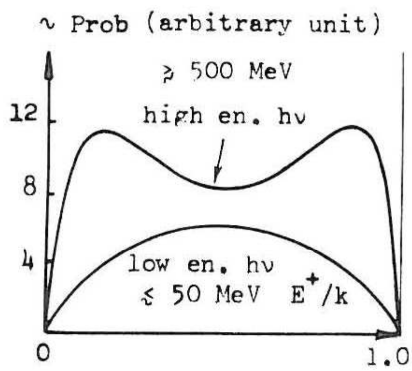
For the angular distribution of pairs, with regard to the incident direction, one comes to the same conclusions as for radiation emission, provided that $Q >> 1$. So

$$\theta_o \simeq \theta_+ \simeq \theta_- \simeq \theta \simeq \frac{1}{Q} \tag{2-2-8}$$



It can also be interesting to find out how the proton energy is shared between electron and positron of the pair. With $Q >> 1$ the general sha of the partial energy taken, say by the positron, is given in figure 2-2/1 For rather low energy quanta, the cross section for pair creation has a broad flat minimum when e^+ and e^- have equal energy. But for high energy, this is replaced by a broad minimum for equal energy and a small maximum when one of the pair receives most of the available energy.

Figure 2-2/1



2-2 c) <u>Relative Importance Between Compton and Pair Production Effects</u>

The total absorption effect (photoelectric effect excepted, assumption verified at involved energies) is the sum of the two partial absorptions: compton effect and pair creation. This can be expressed in the form of exponential decrease for the number of initial incoming photons.

$$N(x) = N(x=o) \quad e^{-\mu(h\nu)x} \qquad (2\text{-}2\text{-}9)$$

Table 2-2/1

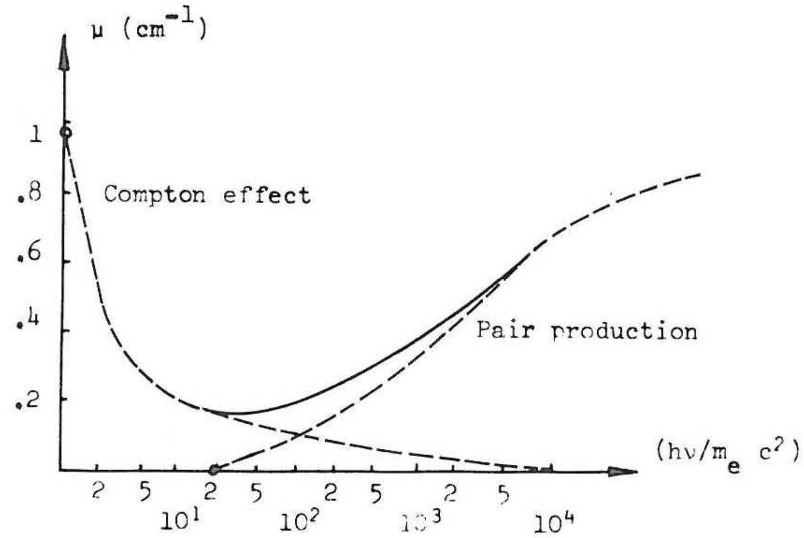
hν / Elt.	10 MeV	40	100	200	600	10^3	5.10^3	10^4	Z
H	3.18	1.60	1.23	1.10	1.10	1.10	1.12	1.12	1
C	1.92	1.31	1.33	1.48	1.66	1.68	1.73	1.73	6
F	2.08	1.70	1.80	1.97	2.20	2.24	2.36	2.37	9
Cl	2.56	2.80	3.22	3.45	3.80	3.90	4.00	4.00	17
Br	3.38	4.65	5.43	5.92	6.45	6.60	6.84	6.85	35

μ-values w
x in gr cm

Some values for elements constituting bubble chamber heavy liquids are give in table (2-2/1)*

In figure (2-2/2), typical curves are given for aluminium corresponding to many effective Z in HLBC conditions.

Figure 2-2/2



* Values interpolated from U.C.R.L. 24-26, Vol. IV (1966).

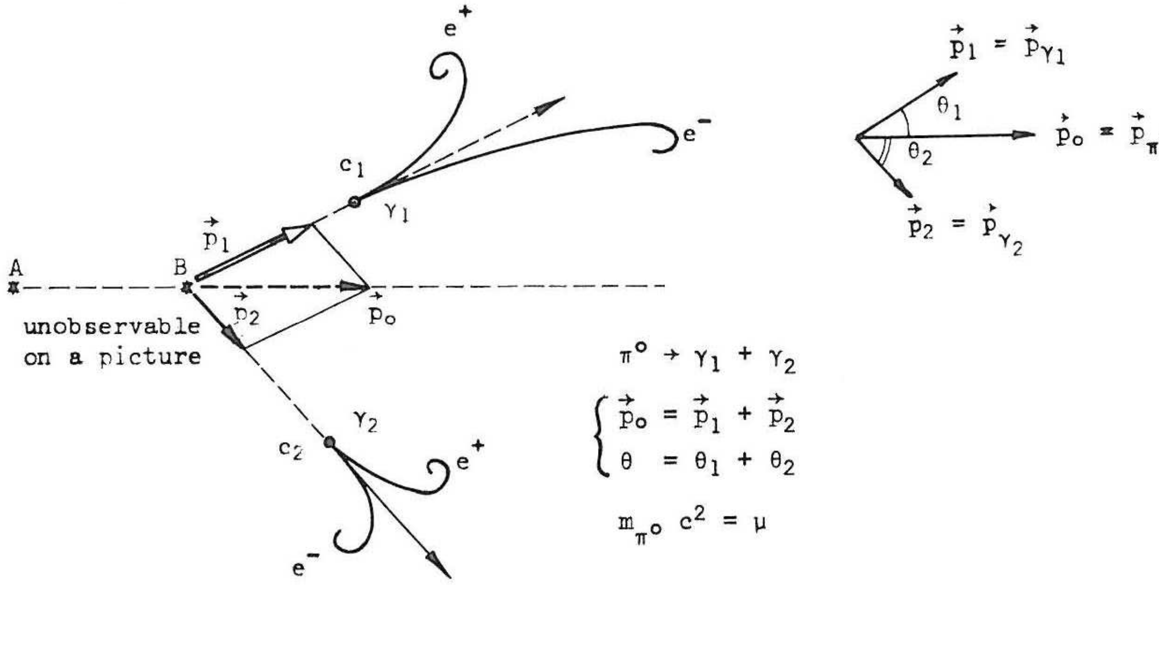
2.2 d) Cascade Shower

In practice, the phenomena due to electrons and photons mix. In fact, the electrons emit photons by radiation, which materialize in turn into electron-positron pairs, which again re-emit photons. This is calle a multiplicative shower or a cascade shower. The initiation can be due t photons as well as to electron or positron.

The study of cascades is out of the limits of these notes. In HLBC, cascade development is, in general, causing trouble for the scannin and the measuring stage. However, one may use the shower to estimate its total energy which is an estimation of the energy of the cascade-initiati particle. (see 3-2 b)

2.3 $\boxed{\pi^o \text{ Disintegration}}$

In HLBC pictures, many photons have their origin from the neutr pi disintegration whose mean life is so short that the path of π^o before disintegration is undetectable. The exact decaying point of the π^o is th known with a good precision.



$$\pi^o \to \gamma_1 + \gamma_2$$

$$\begin{cases} \vec{p}_o = \vec{p}_1 + \vec{p}_2 \\ \theta = \theta_1 + \theta_2 \end{cases}$$

$$m_{\pi^o} c^2 = \mu$$

2.3 a) Kinematics

Energy-momentum conservation law is written as

$$p^2_{\pi^o} = p_o^2 = p_1^2 + p_2^2 + 2p_1p_2 \cos\theta \qquad (2\text{-}3\text{-}1)$$

$$p^2 + p_o^2 = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1p_2 \qquad (2\text{-}3\text{-}2)$$

By elimination it becomes

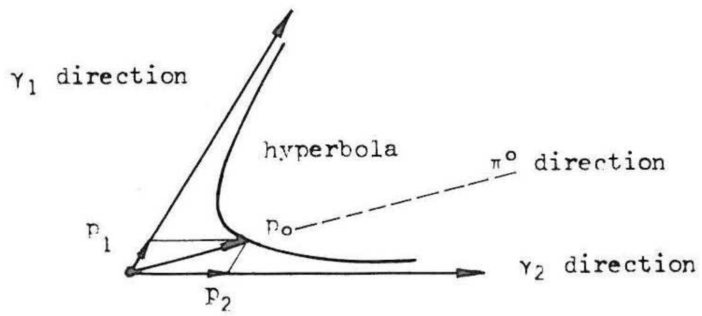
$$2p_1p_2(1 - \cos\theta) = \mu^2 = 4p_1p_2 \sin^2\frac{\theta}{2} \qquad (2\text{-}3\text{-}3)$$

with consequences

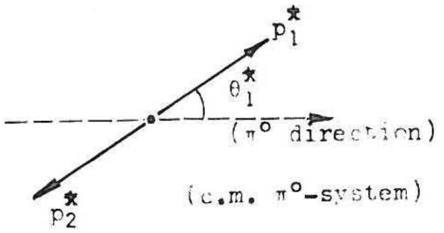
a) the minimum value of θ, θ_{min}, is obtained for $p_1 = p_2 = \frac{E_o}{2}$. In other words, the equipartition of π_o-energy between the two gammas occur when their aperture angle is minimum.

b) For each given θ angle, all possible values for the π^o-momentu are lying on a hyperbola whose equation, in $(\vec{p}_1,\vec{p}_2)$ plane, is

$$p_1\ p_2 = \frac{\mu^2}{4}\ \frac{1}{\sin^2\frac{\theta}{2}} = \text{constant} \qquad (2\text{-}3\text{-}4)$$



What can be said about the p_1 (or p_2) lab.-distribution ? In the c.m. system of the π^o, because its zero spin, the distribution of the two opposite photons is isotropic (θ^* = angle with the incoming direction of the neutral meson).



$p_1^* = p_2^* = \frac{\mu}{2} = E_1^* = E_2^*$ and the numb dN of photons emitted in the θ_1^*-direction is

$$\frac{dN}{d(\cos\theta_1^*)} = k = \text{constant}$$

Going back to the laboratory-system, by Lorentz transform,

$$E_1 = \gamma_o E_1^* + \beta_o \gamma_o p_1^* \cos\theta_1^*$$

with β_o = velocity of π^o in c-unit (lab-system) and $\gamma_o = (1 - \beta_o^2)^{-1/2}$ or

$$E_1 = \frac{1}{2}\left[E_o + p_o \cos\theta_1^*\right]$$

$$E_2 = \frac{1}{2}\left[E_o - p_o \cos\theta_1^*\right]$$

and

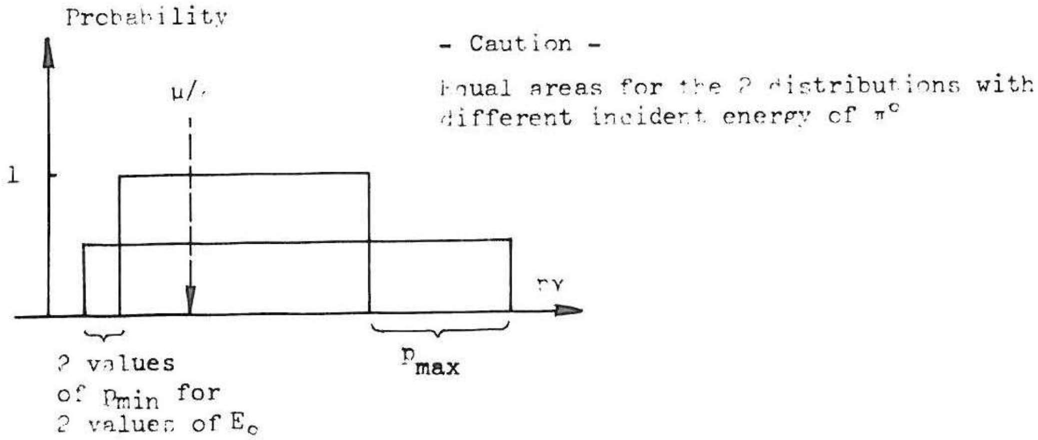
$$\frac{dN}{dE_1} dE_1 = \frac{dN}{d(\cos\theta_1^*)} dE_1 \frac{d(\cos\theta_1^*)}{dE_1} = k \cdot \frac{p_o}{2} dE_1 = K\, dE_1 \quad \text{(2-3-}$$

so that p_1 and p_2 are uniformly distributed between two limits correspond to $\theta_1^* = 0$ or 2π

$$(p_1 \text{ or } p_2)_{min} = \frac{E_o - p_o}{2} \qquad (p_1 \text{ or } p_2)_{max} = \frac{E_o - p_o}{2}$$

including the value $(\mu/2)$. Figure 2-3-1 gives the distribution for p_1 an p_2.

Figure (2-3/1)



2.3 b) Angular Distribution of the 2 Gammas

Because the c.m. isotropy, more γ's are emitted in the unit solid angle centered on $\theta_1^* = \pi/2$. (Let us remark that, in this case, p_2^* is also perpendicular to the incident direction of the π_o) The consequence is that $\theta_1 = \theta_2$ is the most probable angle, then $p_1 = p_2 = p_o/2$. In other words, equipartition of the π_o energy between γ_1 and γ_2 is the most probable repartition and the most symmetrical. By Lorentz transformation application and with

$$\theta = \theta_1 + \theta_2$$

$$\theta^* = \theta_1^* + \theta_2^* = \pi$$

one can deduce

$$\cos\theta = 1 - \frac{2(1-\beta_o^2)}{1-\beta_o^2\cos^2\theta_1^*} \qquad (2\text{-}3\text{-}6)$$

or

$$\cos\theta_1^* = \frac{\left(p_o^2\, tg^2\frac{\theta}{2} - \mu^2\right)^{1/2}}{p_o\, tg\frac{\theta}{2}}$$

leading to

$$\frac{d\cos\theta_1^*}{d\cos\theta_1} = -\frac{1}{4}\,\frac{\left(1+tg^2\frac{\theta}{2}\right)^2}{p_o\, tg^3\frac{\theta}{2}\left(p_o^2\, tg^2\frac{\theta}{2}-\mu^2\right)^{1/2}}$$

so that

$$\frac{dN}{d\theta} = \frac{\mu^2}{8\,p_o\, tg^4\frac{\theta}{2}}\cdot\frac{\left(1+tg^2\frac{\theta}{2}\right)^2}{\left(p_o^2\, tg^2\frac{\theta}{2}-\mu^2\right)^{1/2}} \qquad (2\text{-}3\text{-}5)$$

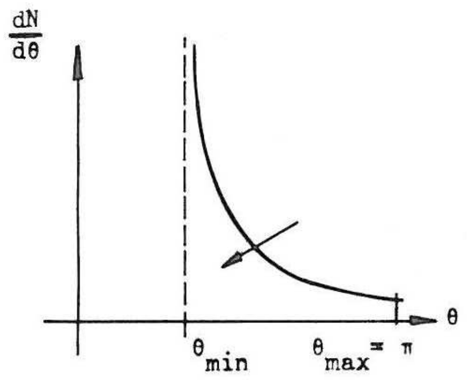
$$\frac{dN}{d\theta} \to \infty \quad \text{for} \quad tg\frac{\theta}{2} = \mu/p_o \quad \text{or} \quad \sin\frac{\theta}{2} = \mu/E_o$$

The formula (2-3-3) can be re-written, when $p_1 = p_2$,

$$\mu^2 = (p_o^2+\mu^2)\,\frac{tg^2\frac{\theta}{2}}{1+tg^2\frac{\theta}{2}} \;\to\; \mu^2 = p_o^2\, tg^2\frac{\theta}{2}$$

So that "The minimum total aperture between the 2 gammas is also the mos probable. It corresponds to the equipartition of the initial energy and of the total angular aperture"

Figure (2-3/2)



$$tg \frac{\theta_m}{2} = \frac{\mu}{p_o}$$

$\frac{dN}{d\theta}$ can never reach zero-value, because the numerator of (2-3-5) i always positive. (see figure 2-3/

In summary, for a given value of p the different configurations (p_1, and θ) are not all equally likely. One may use in π^o-fit, the geometr a priori probability (2-3-5). See § 3.3.

2.4 Implications for H.L.B.C. Pictures

From what we have seen above, if one knows how to measure electrons, it is possible to go back to γ and eventually to π^o or other events more elaborated (such as $K^+ \rightarrow \pi^o + e^+ + \nu$ involving only electronic trajectories). But many spurious γ's and then many spurious electrons are mixed with actually interesting γ or $e^\pm$. These wrong γ's make the sorting more difficult and, besides, are the most frequently annoying. One can yet except total length measurement of the cascade and use of the direction of bremsstrahlung gamma to define the tangent to an electron trajectory.

Radiation emission introduces a new situation which is not met with ordinary tracks. For the latter, all perturbations coming from the medium are supposed small compared to measurement errors. Trajectory maintains its "circular shape". Furthermore, the mean "washing out" is nu

So,one considers that the measured points are identical to those the particles would have passed if it had travelled in vacuum. Spurious effects are taken into account by including them when computing errors. At this level already difficulties appear from the energy loss. The energy loss by collision skews uniformly the trajectory a little.

For electron trajectories, the medium has a catastrophic influenc First, because of the very small value of m_e, scattering becomes very often greater than the measurement uncertainty, but its mean value is still zero. Secondly, the fraction of energy lost by a radiation act can never be recovered afterwards. Then the ideal "circular shape" is never convenient for the actual trajectory, the more because radiation emission obeys a diss metric distribution so that the mean energy loss is not equal to the most probable energy loss. These difficulties are important and their consequen is essentially a rather poor estimation of relevant parameters. They also the error distributions.

One could use the medium itself as a means of measurement as, by the way, emulsionists do with multiple scattering; but this seems out of range of usual possibilities for HLBC's, at least for medium sized chambers. However, with γ and π_o, one has in addition to the ordinary kinematical constraints, "quite pure" geometrical constraints. Indeed, if we look back to formulas (2-1-9) and (2-2-6), we see that for electron or γ energies >> m_e c^2 (and this is generally true), one can consider first, that radiation gives rise only to sudden jumps of the local curvature; secondly for most of them, gammas are made from two initially tangent electron-positron trajector These two geometrical constraints are not always strictly fulfilled, but in practice, because of the scattering and measurement errors, we can consider that they are realized. Some exceptions can occur, but only seldom (especia for angular aperture of γ, but then no correct momentum estimation is possib because the recoil energy is ignored).

In addition, we have already mentioned that, at least in principle if stochastically justified, an a priori geometrical configuration probabili can be used in π_o-reconstruction, simultaneously with ordinary fit constrain